1/17/18

Midterm TOMORROW

⊛ Bring your calculator!!!

Name________________________________ Calculus

Review

This review sheet should NOT serve as your only review. You should review all notes and tests.

Questions 1 through 7 refer to the graph of $y = f(x)$ shown to the right.

1. $\lim_{x \to 1^-} f(x) =$ $-\infty$

2. $\lim_{x \to 1^+} f(x) =$ ∞

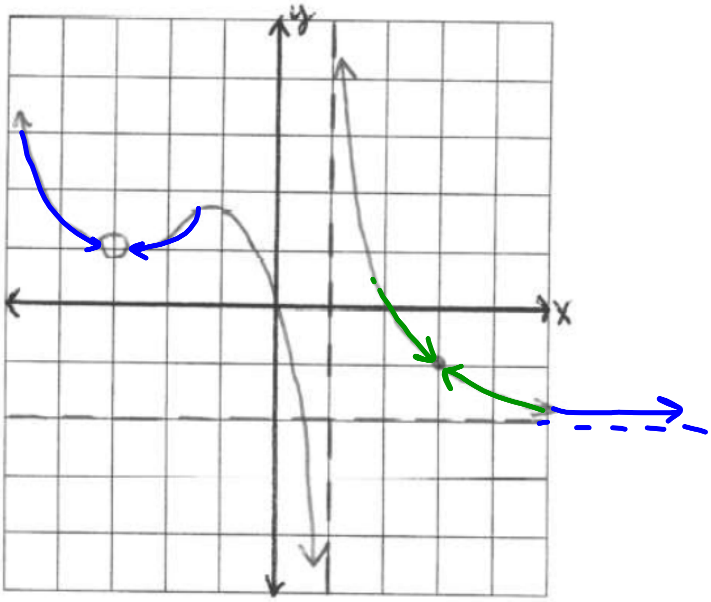
3. $\lim_{x \to 1} f(x) =$ DNE

4. $\lim_{x \to -3} f(x) =$ 1

5. $\lim_{x \to 3} f(x) =$ -1

6. $\lim_{x \to -\infty} f(x) =$ ∞

7. $\lim_{x \to \infty} f(x) =$ -2



For each of the following functions, use the definition of derivative to find $f'(x)$.

Recall: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

8. $f(x) = 2x^2 - 8x + 5$

9. $f(x) = \sqrt{3x+1}$

Find the derivative of each of the following:

10. $f(x) = 5x + 2\sqrt[3]{x} - \frac{3}{x^2}$

11. $f(x) = \sin^2(3x+1)$

12. $f(x) = \ln(\sin x)$

13. $f(x) = \ln\left(\sqrt{2x+3}\right)$

14. $f(x) = \frac{e^{2x}}{x^2}$

15. $f(x) = \sqrt[4]{(x^2+5x)^3}$

16. $f(x) = \sqrt{x}\tan x$

17. $f(x) = x^3 \sec\left(e^{3x} - 1\right)$

18. $f(x) = e^{\sqrt{2x}}$

19. Find the slope of the line tangent to $y = x^2 \ln(3x)$ when $x = 1$.

8)
$$f(x)=2x^2-8x+5 \qquad \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\lim_{h\to 0}\frac{2(x+h)^2-8(x+h)+5-2x^2+8x-5}{h}$$

$$\lim_{h\to 0}\frac{\cancel{2x^2}+4xh+2h^2\cancel{-8x}-8h\cancel{+5}\cancel{-2x^2}\cancel{+8x}\cancel{-5}}{h}$$

$$\lim_{h\to 0}\frac{4xh+2h^2-8h}{h}=\frac{h(4x+2h-8)}{h}=4x+2h-8$$

$$h=0 \;\therefore\; 4x+2(0)-8$$
$$f'(x)=\boxed{4x-8}$$

9) $f(x) = \sqrt{3x+1}$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{\sqrt{3(x+h)+1} - \sqrt{3x+1}}{h} \left(\frac{\sqrt{3(x+h)+1} + \sqrt{3x+1}}{\sqrt{3(x+h)+1} + \sqrt{3x+1}} \right)$$

$$\frac{3(x+h)+1 - (3x+1)}{h\left(\sqrt{3x+3h+1} + \sqrt{3x+1}\right)} = \frac{\cancel{3x}+3h\cancel{+1}\cancel{-3x}\cancel{-1}}{h\left(\sqrt{3x+3h+1} + \sqrt{3x+1}\right)}$$

$$\lim_{h \to 0} \frac{3\cancel{h}}{\cancel{h}\left(\sqrt{3x+3h+1} + \sqrt{3x+1}\right)} = \lim_{h \to 0} \frac{3}{\sqrt{3x\cancel{+3h}+1} + \sqrt{3x+1}}$$

Let $h=0$

$$\boxed{\frac{3}{2\sqrt{3x+1}}}$$

Check:

$$f(x) = \sqrt{3x+1} = (3x+1)^{1/2}$$

$$f'(x) = \frac{1}{2}(3x+1)^{-\frac{1}{2}}(3)$$

$$= \frac{3}{2(3x+1)^{1/2}} = \frac{3}{2\sqrt{3x+1}}$$

10) $f(x) = 5x + 2\sqrt[3]{x} - \frac{3}{x^2}$

$f(x) = 5x + 2x^{1/3} - 3x^{-2}$

$f'(x) = 5 + \frac{2}{3}x^{-2/3} + 6x^{-3}$

OR

$f'(x) = 5 + \frac{2}{3\sqrt[3]{x^2}} + \frac{6}{x^3}$

11) $f(x) = \sin^2(3x+1) = (\sin(3x+1))^2$

$f'(x) = \underline{2}(\sin(3x+1)) \cdot (\cos(3x+1))(\underline{3})$

$= \underline{6}(\sin(3x+1))(\cos(3x+1))$

12) $f(x) = \ln(\sin(x))$

$f'(x) = \frac{1}{\sin(x)} \cdot \cos(x) = \frac{\cos(x)}{\sin(x)} = \boxed{\cot(x)}$

13) $f(x) = \ln(\sqrt{2x+3}) = \ln((2x+3)^{1/2})$

$f'(x) = \frac{1}{\sqrt{2x+3}} \cdot \cancel{\frac{1}{2}}(2x+3)^{-1/2} \cdot \cancel{(2)}$

$= \frac{1}{\sqrt{2x+3}} \cdot (2x+3)^{-1/2}$

$= \frac{1}{\sqrt{2x+3}} \cdot \frac{1}{(2x+3)^{1/2}}$

$= \frac{1}{\sqrt{2x+3}} \cdot \frac{1}{\sqrt{2x+3}} = \frac{1}{2x+3}$

14) $f(x)=\dfrac{e^{2x}}{x^2}$ OR $= e^{2x}\cdot x^{-2}$

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$f'(x)=\dfrac{(x^2)(e^{2x})(2)-(e^{2x})(2x)}{(x^2)^2}$

$f'(x)=\dfrac{2e^{2x}\cancel{x^2}^{\,x}-2e^{2x}\cancel{x}}{\cancel{x^4}_{\,x^3}}$

$\boxed{f'(x)=\dfrac{2xe^{2x}-2e^{2x}}{x^3}}$

15) $f(x)=\sqrt[4]{(x^2+5x)^3}=(x^2+5x)^{3/4}$

$f'(x)=\frac{3}{4}(x^2+5x)^{-1/4}\cdot(2x+5)$

$=\dfrac{3}{4(x^2+5x)^{1/4}}\cdot\dfrac{2x+5}{1}=\boxed{\dfrac{6x+15}{4\sqrt[4]{x^2+5x}}}$

16) $f'(x)=\sqrt{x}\,(\sec^2 x)+\frac{1}{2}x^{-1/2}\cdot\tan x=\sqrt{x}\cdot\sec^2(x)+\dfrac{\tan x}{2\sqrt{x}}$

17) $f'(x)=3x^2\cdot\sec(e^{3x}-1)+x^3\cdot\sec(e^{3x}-1)\tan(e^{3x}-1)(3e^{3x})$

18) $f(x)=e^{\sqrt{2x}}=e^{(2x)^{1/2}}$ $\quad \cancel{\tfrac{1}{2}}(2x)^{-1/2}\cdot\cancel{2}$

$f'(x)=e^{\sqrt{2x}}\cdot\cancel{\frac{1}{2}}(2x)^{-1/2}\cancel{(2)}$

$=\dfrac{e^{\sqrt{2x}}}{(2x)^{1/2}}=\boxed{\dfrac{e^{\sqrt{2x}}}{\sqrt{2x}}}$

19) Plug $x=1$ into the derivative

$y=\overset{f}{x^2}\cdot\overset{g}{\ln(3x)}$ $\quad y'=f'g+fg'$

$y'=(2x)(\ln(3x))+(\cancel{x^2}^{\,x})(\frac{1}{\cancel{3x}})(\cancel{3})$

$y'=(2x)(\ln(3x))+x$

Plug 1 into x $\quad y'=(2(1))(\ln(3(1)))+1$

@$x=1\rightarrow y'=3.197$

20. Write the equation of the line tangent to $y = 3x^2 - 2x + 1$ when $x = -1$.

21. Write the equation of the normal to $y = 5 - x^2$ when $x = 2$.

22. An object moves along a line so that its position at time t is given by $s(t) = 2t^3 - 15t^2 + 24t - 10$ where $t \geq 0$.

a. What is the object's position at time $t = 3$?
b. What is the object's velocity at time $t = 3$?
c. What is the object's acceleration at time $t = 3$?
d. Is the object speeding up or slowing down at $t = 3$? Justify your response.
e. When is the object at rest?
f. When is the object moving right?
g. How far does the object travel in the first 3 seconds?

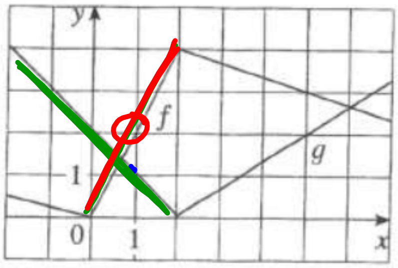
23. If f and g are the functions shown below. Let $h(x) = \boxed{f(g(x))}$ and $s(x) = f(x)g(x)$.

Find: $h'(1)$ and $s'(1)$

$h'(\cancel{x}) = f'(g(\cancel{x})) \cdot g'(\cancel{x})$

$f'(1) \cdot g'(1)$

$(2)(-1) = -2$



24. The following table records the values of $f, f', g, and\ g'$ at $x = 1$, $x = 2$, and $x = 3$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	1	2	3
2	5	4	3	4
3	0	6	-1	-2

$h'(\cancel{x}) = f'(g(\cancel{x})) \cdot g'(\cancel{x})$

$f'(2) \cdot 3$

$4 \cdot 3 = \boxed{12}$

If $\boxed{n(x) = \frac{f(x)}{g(x)}}$, $\boxed{h(x) = f(g(x))}$, find the value of each of the following: a) $n'(2)$ b) $h'(1)$

25. If $f(x) = \sqrt[3]{(x^2 - 2x - 1)^2}$, then $f'(0) =$

$(3x+2)^2$

28. Find $\frac{dy}{dx}$ for the given curve: $x^3 + y^3 = 18y$

29. Find $\frac{dy}{dx}$ for the given curve: $x^2y - xy^2 = 4x$

30. Write the equation of the tangent to $x^2 - xy = y^2 + 1$ in the first quadrant when $y = 1$.

20) Equation we need: Slope (derivative) Point (function)

Slope: @ $x=-1$

$y=3x^2-2x+1$

$y'=6x-2$

$y'=6(-1)-2$

$y'=-8$ ← slope

Point: $x=-1$

$y=3(-1)^2-2(-1)+1$

$y=6$

$(-1,6)$

$$\boxed{y-6=-8(x+1)}$$

21) Point: $x=2$

$y=5-x^2$

$y=5-2^2$

$y=1$

$(2,1)$

Slope: @ $x=2$

$y'=-2x$

$y'=-2(2)$

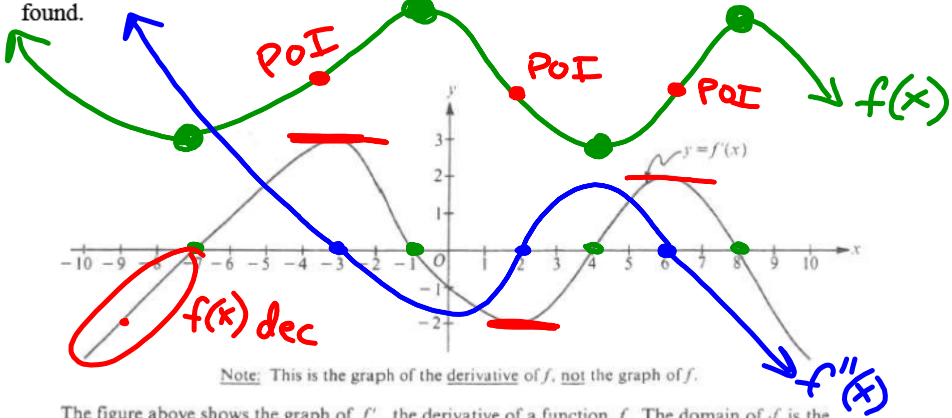
$y'=-4$ ← slope of tangent

$$\boxed{y-1=-4(x-2)}$$

33. Given the function $f(x) = x^4 - 4x^3$, find:

a) the zeros of the function
b) the critical points and the intervals of increasing and decreasing.
c) Any possible inflection points and intervals of concave up or concave down.
d) Finally, sketch the graph. Use your analysis from the 1st and 2nd derivative tests and the zeros you found.

34.



Note: This is the graph of the derivative of f, not the graph of f.

The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-10 \leq x \leq 10$.

(a) For what values of x does the graph of f have a horizontal tangent? $x = -7, -1, 4, 8$

(b) For what values of x in the interval $(-10, 10)$ does f have a relative maximum? Justify your answer. $x = -1$ $x = 8$

(c) For what values of x is the graph of f concave downward?

22) $s(t) = 2t^3 - 15t^2 + 24t - 10$ $\qquad s(3) = -19$ (a)

$v(t) = 6t^2 - 30t + 24$ $\qquad v(3) = -12$ (b)

$a(t) = 12t - 30$ $\qquad a(3) = 6$ (c)

d) Slowing b/c $v(t)$ and $a(t)$ are not same sign

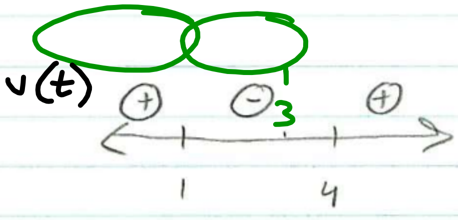
e) $v(t) = 0$

$0 = 6t^2 - 30t + 24$

$0 = 6(t^2 - 5t + 4)$

$0 = (t-4)(t-1)$

$= t = 4 \quad t = 1$



f) $(0,1) \cup (4,\infty)$

g) $|s(0) - s(1)| + |s(1) - s(3)|$

$|-10 - 1| + |1 - (-19)|$

$11 + 20$

31 units

23) $h'(x) = f'(g(x)) \cdot g'(x)$

$h'(1) = f'(g(1)) \cdot g'(1)$

$= f'(1) \cdot (-1)$

$= 2 \cdot (-1) = \boxed{-2}$

$s'(x) = f(x)g'(x) + f'(x)g(x)$

$= f(1)g'(1) + f'(1)g(1)$

$= (2)(-1) + (2)(1)$

$= -2 + 2$

$= \boxed{0}$

24) a) $r'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

$r'(2) = \dfrac{g(2)f'(2) - f(2)g'(2)}{(g(2))^2} = \dfrac{(3)(4) - (5)(4)}{3^2} = \dfrac{12-20}{9} = \boxed{\dfrac{-8}{9}}$

b) $h'(x) = f'(g(x)) \cdot g'(x)$

$= f'(g(1)) \cdot g'(1)$

$= f'(2) \cdot g'(1)$

$= 4 \cdot 3$

$= \boxed{12}$

25) $f(x) = \sqrt[3]{(x^2-2x-1)^2} = (x^2-2x-1)^{2/3}$

$f'(x) = \frac{2}{3}(x^2-2x-1)^{-1/3} \cdot (2x-2)$

$= \frac{2(2x-2)}{3\sqrt[3]{x^2-2x-1}} = \frac{4x-4}{3\sqrt[3]{x^2-2x-1}}$

$f'(0) = \frac{4(0)-4}{3\sqrt[3]{0^2-2(0)-1}} = \frac{-4}{3(-1)} = \frac{-4}{-3} = \boxed{\frac{4}{3}}$

28) $x^3+y^3=18y$ $\quad 3x^2+3y^2\frac{dy}{dx} = 18\frac{dy}{dx}$

$3x^2 = 18\frac{dy}{dx} - 3y^2\frac{dy}{dx}$

$3x^2 = (18-3y^2)\frac{dy}{dx}$

$\frac{3x^2}{18-3y^2} = \frac{dy}{dx} = \boxed{\frac{x^2}{6-y^2}}$

29) $x^2y - xy^2 = 4x$

$x^2\frac{dy}{dx} + 2xy - \left(x2y\frac{dy}{dx} + y^2\right) = 4$

$x^2\frac{dy}{dx} + 2xy - 2xy\frac{dy}{dx} - y^2 = 4$

$\frac{dy}{dx}(x^2 - 2xy) = 4 - 2xy + y^2$

$$\frac{dy}{dx} = \boxed{\frac{4-2xy+y^2}{x^2-2xy}}$$

30) $x^2 - xy = y^2 + 1$ when $y=1$

Point: $x^2 - x(1) = 1^2 + 1$

$x^2 - x = 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ $x = -1$ reject not in QI

Point: $(2,1)$

Slope: $2x - x\frac{dy}{dx} - y = 2y\frac{dy}{dx} + x\frac{dy}{dx}$

$2x - y = \frac{dy}{dx}(2y + x)$

$\frac{2x-y}{2y+x} = \frac{dy}{dx}$

$\frac{2(2)-1}{2(1)+2} = \frac{3}{4}$ ← slope @ $(2,1)$

$$\boxed{y - 1 = \frac{3}{4}(x-2)}$$

33) $f(x) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2$

$f(x)=0$

a) $0 = x^4 - 4x^3$

$0 = x^3(x-4)$

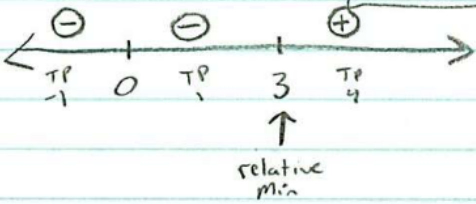
$\boxed{x=0 \quad x=4}$

b) $f'(x)=0$

$0 = 4x^3 - 12x^2$

$0 = 4x^2(x-3)$

$x=0 \quad x=3$



Increasing: $(3, \infty)$

Decreasing: $(-\infty, 0) \cup (0, 3)$

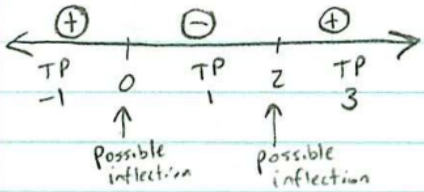
$f''(x)=0$

c) $f''(x) = 12x^2 - 24x$

$0 = 12x(x-2)$

$x=0 \quad x=2$

$f''(x)$

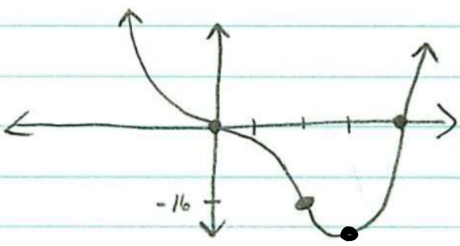


Concave Up: $(-\infty, 0) \cup (2, \infty)$

Concave Down: $(0, 2)$

Inflection Pts: $(0,0)$ and $(2, -16)$

d)



Remember the graph is $f'(x)$ The DERIVATIVE!

34) a) Where $f'(x) = 0$

$x = -7, -1, 4, 8$

b) Where $f'(x)$ goes from $(+)$ to $(-)$

$x = -1$ $x = 8$

c) When $f'(x)$ is decreasing

$(-3, 2) \cup (6, \infty)$

$f(x)$

$f'(x) = g(x)$

$f''(x) = g'(x)$

Derivative Rules

$y = \ln(u)$ $\qquad \frac{dy}{dx} = \frac{1}{u} \cdot u'$

"u" is a function of x

$y = e^u$ $\qquad \frac{dy}{dx} = e^u \cdot u'$

"u" is a function of x

$y = f(g(x))$ $\qquad \frac{dy}{dx} = f'(g(x)) \cdot g'(x)$

$y = \csc(3x)$

$y' = -\csc(3x) \cdot \cot(3x)(3)$

$y = x^2 \cdot \csc(3x)$

$y' = 2x \cdot \csc(3x) + x^2 \cdot \left(-\csc(3x) \cdot \cot(3x)(3)\right)$

Absolute Max/Min on interval

1) Set derivative $= 0$ and solve

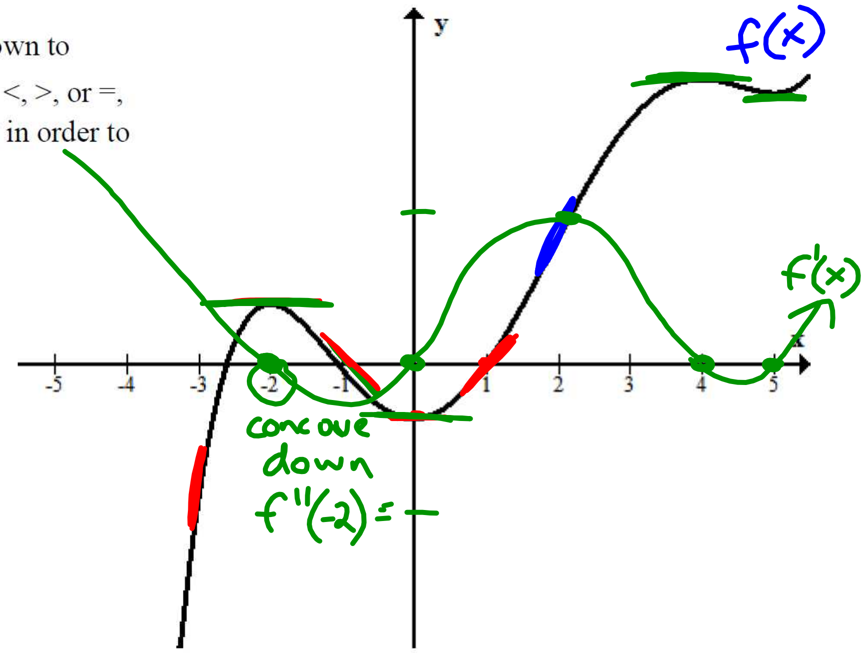
Plug into original

2) Plug in those values that you find in Step 1 that are in interval

3) Plug in the interval values as well

4) The Max/Min is the value you get, NOT the entire point!

4. Given the graph of $y = f(x)$ shown to the right. Fill in the blank with <, >, or =, in each of the statements below in order to create a true statement.



a) $f'(2)$ __$>$__ 0 (+   0)

b) $f'(-2)$ __$=$__ 0 (0   0)

c) $f'(-1)$ __$<$__ 0 (−   0)

d) $f''(-2)$ __$<$__ 0 (−   0)

e) $f''(0)$ __$>$__ $f(1)$ (+   0)

f) $f'(-3)$ __$>$__ $f'(1)$

g) $f'(0)$ __$>$__ $f'(-1)$ (0   −)

h) $f'(0)$ __$=$__ $f(1)$ (0   0)